Math 135 Lab 3 – Binomial Distribution and Central Limit Theorem

**Binomial distribution investigation:**

For this section you will use StatCrunch in MML. Use the following menu items: STAT->Calculators->Binomial. With the Binomial distribution you are to use the options in the following table. As you are changing the values please note:

1. The shape of the curve as you, change N and keep P constant.
2. The shape of the curve as you, change P and keep N constant.
3. Complete the table below to help track the patterns. X and y coordinates of peaks can be approximate.

|  |  |  |  |
| --- | --- | --- | --- |
| **P** | **N** | **Peak**  **(x, y)** | **Shape**  Symmetrical, left-skewed or right skewed |
| .5 | 5 |  |  |
| .5 | 10 |  |  |
| .5 | 20 |  |  |
| .5 | 40 |  |  |
| .5 | 80 |  |  |
| .5 | 160 |  |  |
| .5 | 320 |  |  |
| .2 | 5 |  |  |
| .2 | 10 |  |  |
| .2 | 20 |  |  |
| .2 | 40 |  |  |
| .2 | 80 |  |  |
| .2 | 160 |  |  |
| .2 | 320 |  |  |
| .8 | 5 |  |  |
| .8 | 10 |  |  |
| .8 | 20 |  |  |
| .8 | 40 |  |  |
| .8 | 80 |  |  |
| .8 | 160 |  |  |
| .8 | 320 |  |  |

Describe the changes in the distribution as P and N change.

1. As P changed from .5 to .2 to .8, how did the shape of the distribution change?
2. As N increased, how did the shape of the distribution change?

**Central Limit Theorem:**

There are two parts of the Central Limit Theorem that are important. One that indicates that no matter what distribution you start with, the sample means approach a normal distribution as the sample size increases. A second is that the standard deviation of the sample means will be σ/sqrt(n). The first part of this lab should help verify that the first part (the sample means approach a normal distribution as the sample size increases) for a binomial distribution. To prove the second part is difficult we can at least get an idea if it works with the following problems.

1. Start with a very simple uniform distribution (1, 2, 3, 4, 5) and do the following:
   1. Find the mean and the standard deviation of the five numbers.
   2. Take all the samples of size 2 (n=2) which includes the samples:
      1. 1,2 mean = \_\_\_\_\_\_\_
      2. 1,3 mean = \_\_\_\_\_\_\_
      3. 1,4 mean = \_\_\_\_\_\_\_
      4. 1,5 mean = \_\_\_\_\_\_\_
      5. 2,3 mean = \_\_\_\_\_\_\_
      6. 2,4 mean = \_\_\_\_\_\_\_
      7. 2,5 mean = \_\_\_\_\_\_\_
      8. 3.4 mean = \_\_\_\_\_\_\_
      9. 3,5 mean = \_\_\_\_\_\_\_
      10. 4,5 mean = \_\_\_\_\_\_\_

Find the mean of each sample of 2 and then the mean and standard deviations of all the 10 sample means. What do you get?

* 1. Take all the samples of size 4 (n=4) which includes the samples:
     1. 1,2,3,4 mean=\_\_\_\_\_\_
     2. 1,3,4,5 mean=\_\_\_\_\_\_
     3. 1,2,4,5 mean=\_\_\_\_\_\_
     4. 1,2,3,5 mean=\_\_\_\_\_\_
     5. 2,3,4,5 mean=\_\_\_\_\_\_

Find the mean of each sample of size and then the mean and standard deviations of all the 5 sample means. What do you get?

* 1. What do you notice about the means for parts a-c?
  2. What do you notice about the standard deviations for parts a-c?

1. Choose the following options in StatCrunch to launch the Central Limit Theorem simulation feature:

Applets->Sampling Distributions

* Set the population distribution to Uniform and with a Lower Bound of 0 and an Upper bound of 50 and press the Compute button.
* Use the options on the dialog box to set the sample size (n) and the samples to the values in the table below.
* Fill out the mean and standard deviations for the four distributions below.

Population mean \_\_\_\_\_\_\_\_, population standard deviation\_\_\_\_\_\_\_

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Distribution | Samples | n | Mean | St Dev |
| Uniform | 5 | 2 |  |  |
| Uniform | 5 | 25 |  |  |
| Uniform | 1000 | 2 |  |  |
| Uniform | 1000 | 25 |  |  |

* Edit the Options and set the population distribution to Right Skewed and press the Compute button.
* Use the options on the dialog box to set the sample size (n) and the samples to the values in the table below.
* Fill out the mean and standard deviations for the four distributions below.

Population mean \_\_\_\_\_\_\_\_, population standard deviation\_\_\_\_\_\_\_

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Distribution | Samples | n | Mean | St Dev |
| Skewed | 5 | 2 |  |  |
| Skewed | 5 | 25 |  |  |
| Skewed | 1000 | 2 |  |  |
| Skewed | 1000 | 25 |  |  |

* Edit the Options and set the population distribution to Bell Shaped (Normal) with a Mean of 25 and a Standard Deviation of 5. Press the Compute button.
* Use the options on the dialog box to set the sample size (n) and the samples to the values in the table below.
* Fill out the mean and standard deviations for the four distributions below.

Population mean \_\_\_\_\_\_\_\_, population standard deviation\_\_\_\_\_\_\_

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Distribution | Samples | n | Mean | St Dev |
| Normal | 5 | 2 |  |  |
| Normal | 5 | 25 |  |  |
| Normal | 1000 | 2 |  |  |
| Normal | 1000 | 25 |  |  |

* Edit the Options and set the population distribution to Continuous Custom with a Lower Bound of 0 and an Upper Bound of 50. Press the Compute button.
* You will need to use your mouse to set up something that looks like a Bimodal Distribution in the Population area.
* Use the options on the dialog box to set the sample size (n) and the samples to the values in the table below.
* Fill out the mean and standard deviations for the four distributions below.

Population mean \_\_\_\_\_\_\_\_, population standard deviation\_\_\_\_\_\_\_

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Distribution | Samples | n | Mean | St Dev |
| Bimodal | 5 | 2 |  |  |
| Bimodal | 5 | 25 |  |  |
| Bimodal | 1000 | 2 |  |  |
| Bimodal | 1000 | 25 |  |  |

* Edit the Options and set the population distribution to Continuous Custom with a Lower Bound of 0 and an Upper Bound of 50. Press the Compute button.
* You will need to use your mouse to set up a random distribution (any distribution you want) in the Population area.
* Use the options on the dialog box to set the sample size (n) and the samples to the values in the table below.
* Fill out the mean and standard deviations for the four distributions below.

Population mean \_\_\_\_\_\_\_\_, population standard deviation\_\_\_\_\_\_\_

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Distribution | Samples | n | Mean | St Dev |
| Yours | 5 | 2 |  |  |
| Yours | 5 | 25 |  |  |
| Yours | 1000 | 2 |  |  |
| Yours | 1000 | 25 |  |  |

1. Describe the change in the distribution, mean, standard deviation, and skew as:
   1. The number of sample changed
   2. The sample size (n) changed
   3. The distribution changed
2. Is there a pattern that is consistent across all distributions?